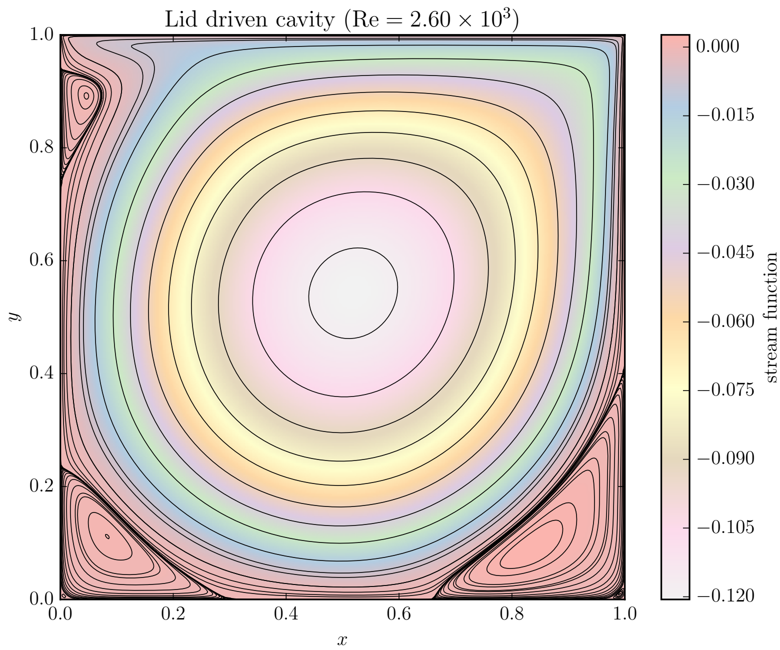
FC1.5 Numerical methods



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Objectives

During the mathematical modeling of a physical problem chances are that one will obtain differential equations, either Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE). In astrophysics the need for numerical simulations is even more critical since the objects under study are far from being located in the lab. A large variety of physical problems are described in that way such as e.g. fluid dynamics, heat conduction, n-body simulation, radiative transfer, etc. Solving the equations of mathematical physics numerically is a good playground to face the classical problems of numerical analysis such as interpolating, integrating, solving linear system of equations, etc . . . . The student will learn how to solve numerical problems focused on the solution of differential equations, using various techniques from numerical analysis, such as the finite difference method. He/she will learn how to build and study the behavior of various numerical schemes used to solve the classical equation of mathematical physics.

Evaluation

Homeworks (10%), written exam (40%) and project (50%)

Main progression steps

- First half of the period : theoretical courses (exam at end term).
- Second half of the period : numerical project and practical works.

Bibliography & Resources

- A First Course in Numerical Analysis, Ralston, A. and Rabinowitz, P., Dover Publications, Inc., 2001
- Finite difference methods for differential equations, LeVeque, R.J., 2007
- Finite difference and spectral methods for ordinary and partial differential equations, Trefethen, L.N., 1996
- Numerical methods for engineers and scientists, Hoffman, Joe D and Frankel, Steven, CRC press, 2001
- Finite volume methods for hyperbolic problems, LeVeque, Randall J, Cambridge university press, 2002

Contents

Part 1: Fundamentals

by G. Niccolini

1. Round-off error and truncation errors.
2. Interpolation
3. Discrete Fourier Transform (DFT)
4. Integration of functions
5. Solving Linear systems of equations
   (a) Iterative methods for sparse systems (Jacobi, Gauss-Seidel, Successive Over-Relaxation, conjugate gradient, . . .)
6. Finite Difference method

Part 2: Partial/Ordinary Differential Equations

by G. Niccolini

1. Numerical solution of ODEs
   (a) Solving differential equation numerically
   - Mathematical properties
   - One-step methods (Euler, RK, Taylor series, explicit vs implicit methods)
   - Multi-step methods (Leap-Frog, Adams-Moulton, Adams-Bashforth, . . .)
   - Stability analysis : notion of zero and absolute stability, characteristic polynomials

   (a) Solving PDEs
   - Mathematical properties
   - PDEs as vector ODEs
   - Boundary conditions
   - Truncation error and consistency
   - Convergence and stability
   - Von Neumann stability criterion

Part 3: Practical works

by G. Niccolini

1. Short intro to programming with `C++` and `python`
2. Development of computer programs to solve linear PDE of mathematical physics (wave equation, Schrödinger, diffusion, advection, . . .)